ASYMPTOTIC INVESTIGATION OF SEEPAGE FLOWS IN THE CASE OF A POWER RESISTANCE LAW

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An asymptotic investigation of the problem of seepage under a dam in a layer of ground of finite thickness is described. The solution and the physical characteristics of the flow are presented.

We will consider plane seepage flow under a dam in a layer of ground of finite power R with a power seepage law

$$\operatorname{grad} H = -\Phi(w)\cdot \mathbf{w}/w,\ \ \Phi(w) = w^k,\ \ k > 0. \tag{1}$$

After mapping the plane of flow in the plane of the hodograph (w, θ) the problem reduces to solving the equation

$$w^2 H_{ww} + (2-k)\, wH_w + kH_{\theta\theta} = 0 \tag{2}$$

in the half strip $0 \leqslant w < \infty$, $-\pi/2 \leqslant \theta \leqslant 0$ with the boundary conditions

$$\begin{gathered} H(w,\ -\pi/2) = H_1 \quad (0 \leqslant w < \infty);\ H(w,\ 0) = (H_1 + H_2)/2 (a \leqslant w \leqslant b); \\ H_\theta(w,\ 0) = 0\ \ (0 \leqslant w \leqslant a,\ b \leqslant w < \infty). \end{gathered} \tag{3}$$

We will introduce as the new unknown

$$h(w,\ \theta) = \frac{2w^{-\alpha}[H(w,\ \theta) - H_1]}{H_2 - H_1},\quad \alpha = (k-1)/2. \tag{4}$$

For $h(w, \theta)$ we have the problem

$$\begin{gathered} w^2 h_{ww} + wh_w + kh_{\theta\theta} - \alpha^2 h = 0, \\ h(w,\ -\pi/2) = 0,\ h_\theta(w,\ 0) = 0\ \ (0 \leqslant w \leqslant a,\ b \leqslant w < \infty), \\ h(w,\ 0) = w^{-\alpha} = f(w) \quad (a \leqslant w \leqslant b). \end{gathered} \tag{5}$$

Applying a Mellin transform with respect to the variable w and parameter s to problem (5), we obtain for $h^*(s, \theta)$ a differential equation, by solving which, taking into account the first conditions of (5), we obtain

$$h^*(s,\ \theta) = A(s) \sin \xi (\theta + \pi/2),\quad \xi = \sqrt{(s^2 - \alpha^2)/k}, \tag{6}$$

$$h^*(s,\ \theta) = \int_0^\infty h(w,\ \theta)\, w^{s-1} dw. \tag{7}$$

We will denote by $f_+(w)$ and $f_-(w)$ the unknown quantities $h(w, \theta)$ when $\theta = 0$, $w > b$ and $w < a$, respectively. Then, to obtain A(s) from the boundary conditions (5), we have

$$\begin{gathered} A(s) \sin(\pi\xi/2) = \varphi_-(s) + \varphi_+(s) + \varphi_0(s);\quad \varphi_+(s) = \int_b^\infty f_+ w^{s-1} dw, \\ \varphi_-(s) = \int_0^a f_- w^{s-1} dw,\ \ \varphi_0(s) = \int_a^b f w^{s-1} dw = (b^{s-\alpha} - a^{s-\alpha})/(s-\alpha), \end{gathered} \tag{8}$$

$$A(s)\, \xi \cos(\pi\xi/2) = \int_a^b h_\theta(w,\ 0) w^{s-1} dw = X(s). \tag{9}$$

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In relations (8) and (9), $X(s)$ and $\varphi_0(s)$ are integer functions of the order of increase of $\exp s\mu$, $\mu = \max|\ln a, \ln b|$; φ_+ is a function that is analytical when $\mathrm{Re}\, s < \alpha + \varepsilon (\varepsilon > 0)$; φ_- is a function that is analytical when $\mathrm{Re}\, s > \alpha - \varepsilon$, and both functions decrease in the corresponding half planes. Functions with a similar analytical structure will henceforth be called "plus" and "minus" functions, respectively. Eliminating $A(s)$ from (8) and (9), we obtain the equation

$$[\varphi_-(s) + \varphi_+(s) + \varphi_0(s)]\, \xi \operatorname{ctg}(\pi\xi/2) = X(s). \tag{10}$$

It contains three unknown functions φ_-, φ_+, and X. Nevertheless, it is sufficient to determine all these functions, taking into account their analytical structure. Multiplying Eq. (10) by $b^{\alpha-s}$, we have

$$[Y_-(s) + Y_+(s) + H_-(s)]\, K(s) = Z_-(s). \tag{11}$$

Here $Y_-(s) = b^{\alpha-s}\varphi_-(s)$, $H_-(s) = b^{\alpha-s}\varphi_0(s)$, $Z_-(s) = b^{\alpha-s}X(s)$ are minus functions, while $Y_+(s) = b^{\alpha-s}\varphi_+(s)$ is a plus function. The coefficient $K(s)$ can be factorized in the form

$$K(s) = \xi \operatorname{ctg}(\pi\xi/2) = K_+(s)\, K_-(s),$$
$$K_\pm(s) = \sqrt{(\alpha/\sqrt{k} \operatorname{cth}(\pi\alpha/2\sqrt{k})} \prod_{n=1}^{\infty} [1 \mp s/\sqrt{\alpha^2 + k(2n-1)^2}] \Big/ [1 \mp s/\sqrt{\alpha^2 + 4kn^2}]. \tag{12}$$

Here $K_+(s)$ and $K_-(s)$ are plus and minus functions, respectively. From (11) and (12) we obtain

$$Y_- K_+ + Y_+ K_+ + H_- K_+ = Z_-/K_-. \tag{13}$$

Similarly, multiplying (10) by $a^{\alpha-s}$ and assuming that

$$Y^*_- = a^{\alpha-s}\varphi_-,\ H^*_+ = a^{\alpha-s}\varphi_0,\ Z^*_+ = a^{\alpha-s}X,\ Y^*_+ = a^{\alpha-s}\varphi_+, \tag{14}$$

we obtain

$$Y^*_- K_- + Y^*_+ K_- + H^*_+ K_- = Z^*_+/K_+. \tag{15}$$

It is obvious that the principal aspect is that the integer functions φ_0 and X, after multiplication by $b^{\alpha-s}$, are minus functions, and after multiplication by $a^{\alpha-s}$ are plus functions, whereas the class of functions φ_- and φ_+ do not change. It is also clear that

$$Y_\pm = d^{\alpha-s} Y^*_\pm,\ H_- = d^{\alpha-s} H^*_+,\ Z_- = d^{\alpha-s} Z^*_+\ (d = b/a,\ g = a/b). \tag{16}$$

Equations (13) and (15) hold in the band $|\mathrm{Re}(s - \alpha)| < \varepsilon$. We will represent them in the form

$$Y_+ K_+ + L_+ + D_+ = Z_-/K_- - L_- - D_-, \tag{17}$$
$$Y^*_- K_- + L^*_- + D^*_- = Z^*_+/K_+ - L^*_+ - D^*_+. \tag{18}$$

Here we will use the expansion of the functions Y_-K_+, H_-K_+, $Y^*_+K_-$, $H^*_+K_-$, that are analytical in the band $|\mathrm{Re}(s - \alpha)| < \varepsilon$, in a sum of the plus and minus functions

$$Y_- K_+ = L_+ + L_-,\ H_- K_+ = D_+ + D_-,$$
$$Y^*_+ K_- = L^*_+ + L^*_-,\ H^*_+ K_- = D^*_+ + D^*_-, \tag{19}$$

which can be done using integrals of the Cauchy type [1, 2]

$$L_\pm = \pm \frac{1}{2\pi i} \int_{\alpha\pm\delta-i\infty}^{\alpha\pm\delta+i\infty} \frac{b^{\alpha-\zeta} K_+(\zeta)\, \varphi_-(\zeta)}{\zeta - s}\, d\zeta,$$
$$L^*_\pm = \pm \frac{1}{2\pi i} \int_{\alpha\pm\delta-i\infty}^{\alpha\pm\delta+i\infty} \frac{a^{\alpha-\zeta} K_-(\zeta)\, \varphi_+(\zeta)}{\zeta - s}\, d\zeta,$$
$$D_\pm = \pm \frac{1}{2\pi i} \int_{\alpha\pm\delta-i\infty}^{\alpha\pm\delta+i\infty} \frac{(1 - g^{\zeta-\alpha})\, K_+(\zeta)}{(\zeta - s)(\zeta - \alpha)}\, d\zeta,$$
$$D^*_\pm = \pm \frac{1}{2\pi i} \int_{\alpha\pm\delta-i\infty}^{\alpha\pm\delta+i\infty} \frac{(d^{\zeta-\alpha} - 1)\, K_-(\zeta)}{(\zeta - s)(\zeta - \alpha)}\, d\zeta$$
$$(0 < \delta < \varepsilon). \tag{20}$$

The existence of the integrals is ensured by the behavior of the functions $K_{\pm}(s)$ ($\sim\sqrt{s}$) at infinity, and by a priori estimate of the behavior of the unknown functions φ_{+} and φ_{-} (being Mellin transforms of bounded functions, φ_{+} and φ_{-} approach zero as $s \to \infty$). Equations (17) and (18) are Wiener–Hopf equations, in view of which the functions

$$\begin{aligned} &P(s)=Y_{+}K_{+}+L_{+}+D_{+}, \ \operatorname{Re} s<\alpha+\delta;\\ &P(s)=Z_{-}/K_{-}-L_{-}-D_{-}, \ \operatorname{Re} s>\alpha-\delta,\\ &P^{*}(s)=Y_{-}^{*}K_{-}+L_{-}^{*}+D_{-}^{*}, \ \operatorname{Re} s>\alpha-\delta;\\ &P^{*}(s)=Z_{+}^{*}/K_{+}-L_{+}^{*}-D_{+}^{*}, \ \operatorname{Re} s<\alpha+\delta \end{aligned} \tag{21}$$

are analytic integer functions of s. It is easy to show that they decrease at infinity and hence, in view of Liouville's theorem, are identically equal to zero. In fact, it follows from estimates for $K_{\pm}$ and $\varphi_{\pm}$ that $L_{\pm} \to 0$ as $|s| \to \infty$ and $D_{\pm} \to 0$ as $|s| \to \infty$. Finally, we have

$$Y_{+}=b^{\alpha-s}\int_{b}^{\infty} f_{+}w^{s-1}dw \leqslant b^{-1}\int_{b}^{\infty} H(w)\,(w/b)^{s-\alpha-1}dw \simeq O(s^{-1}),$$

so that $Y_{+}K_{+} \to 0$ as $|s| \to \infty$. Hence it follows that $P(s) \equiv 0$. It can be shown similarly that $P^{*}(s) \equiv 0$. Hence, we have from (17), (18), and (21)

$$Y_{+}=-(L_{+}+D_{+})/K_{+}, \quad Y_{-}^{*}=-(L_{-}^{*}+D_{-}^{*})/K_{-} \tag{22}$$

or, reverting to the functions φ_{+} and φ_{-}

$$\varphi_{+}(s)=-\frac{b^{s-\alpha}}{K_{+}(s)}\left[\frac{1}{2\pi i}\int_{\alpha+\delta-i\infty}^{\alpha+\delta+i\infty}\frac{b^{\alpha-\zeta}K_{+}(\zeta)\,\varphi_{-}(\zeta)}{\zeta-s}\,d\zeta+\frac{1}{2\pi i}\int_{\alpha+\delta-i\infty}^{\alpha+\delta+i\infty}\frac{(1-g^{\zeta-\alpha})\,K_{+}(\zeta)d\zeta}{(\zeta-s)\,(\zeta-\alpha)}\right], \tag{23}$$

$$\varphi_{-}(s)=\frac{a^{s-\alpha}}{K_{-}(s)}\left[\frac{1}{2\pi i}\int_{\alpha-\delta-i\infty}^{\alpha-\delta+i\infty}\frac{a^{\alpha-\zeta}K_{-}(\zeta)\,\varphi_{+}(\zeta)}{\zeta-s}\,d\zeta+\frac{1}{2\pi i}\int_{\alpha-\delta-i\infty}^{\alpha-\delta+i\infty}\frac{(d^{\zeta-\alpha}-1)\,K_{-}(\zeta)\,d\zeta}{(\zeta-s)\,(\zeta-\alpha)}\right]. \tag{24}$$

Relations (23) and (24) are integral equations for the analytical functions φ_{+} and φ_{-}. We will solve these equations for small values of the parameter g, which corresponds to the asymptotic of a heavy layer of ground. We will first consider the expression

$$\eta_{0}(s)=\frac{1}{2\pi i}\int_{\alpha+\delta-i\infty}^{\alpha+\delta+i\infty}\frac{(1-g^{\zeta-\alpha})\,K_{+}(\zeta)}{\zeta-s}\,d\zeta. \tag{25}$$

The contribution to the integral from the first term in the square brackets is

$$K_{+}(s)/(s-\alpha)+K_{+}(\alpha)/(\alpha-s)].$$

To calculate the contribution from the second term we note that the function $g^{\zeta-\alpha}$ is analytical in the right half plane of ζ and decreases as $\exp(\zeta \ln g)$. On the other hand, $K_{+}(\zeta)$ extends into the right half plane as a meromorphic function, which follows from the representation

$$K_{+}(s)=K(s)/K_{-}(s)=\xi\operatorname{ctg}(\pi\xi/2)/K_{-}(s) \tag{26}$$

and the regularity of $K_{-}(s)$ in the right half plane. Denoting the poles and residues of this function by s_{n}^{+} and r_{n}^{+}, we have

$$s_{n}^{+}=\sqrt{\alpha^{2}+4kn^{2}}; \quad r_{n}^{+}=\left[\frac{\alpha}{\sqrt{k}}\operatorname{cth}(\pi\alpha/2\sqrt{k})\right]^{-1/2}\frac{8n^{2}k}{\pi s_{n}^{+}}\prod_{j=1}^{\infty}\frac{1+s_{n}^{+}(\alpha^{2}+4kj^{2})^{-1/2}}{1+s_{n}^{+}[\alpha^{2}+k(2j-1)^{2}]^{-1/2}}. \tag{27}$$

Then, transferring the contour of integration successively to the right we obtain

$$-\frac{1}{2\pi i}\int_{\alpha+\delta-i\infty}^{\alpha+\delta+i\infty}\frac{g^{\zeta-\alpha}K_{+}(\zeta)}{(\zeta-s)\,(\zeta-\alpha)}\,d\zeta=\sum_{n=1}^{\infty}\frac{g^{s_{n}^{+}-\alpha}r_{n}^{+}}{(s_{n}^{+}-s)(s_{n}^{+}-\alpha)}. \tag{28}$$

It is obvious that as $g \to 0$ the whole of this expression approaches zero as

$$g^{s_{1}^{+}-\alpha}r_{1}^{+}/[(s_{1}^{+}-s)(s_{1}^{+}-\alpha)].$$

Hence,

$$\eta_{0}(s)=[K_{+}(s)-K_{+}(\alpha)]/(s-\alpha)+\sum_{n=1}^{\infty}g^{s_{n}^{+}-\alpha}r_{n}^{+}/[(s_{n}^{+}-s)\,(s_{n}^{+}-\alpha)]. \tag{29}$$

On the other hand, it is easy to obtain for the first term of Eq. (23)

$$I_1=\frac{1}{2\pi i}\int_{\alpha+\delta-i\infty}^{\alpha+\delta+i\infty}\frac{b^{\alpha-\zeta}K_+(\zeta)\,\varphi_-(\zeta)}{\zeta-s}\,d\zeta=-\sum_{n=1}^{\infty}\frac{b^{\alpha-s_n^+}\varphi_-(s_n^+)\,r_n^+}{s_n^+-s}\,. \tag{30}$$

But for real positive s

$$|\varphi_-(s)|=\left|\int_0^a h(w)w^{s-1}\,dw\right|=\left|\frac{2}{H_2-H_1}\int_0^a[H(w)-H_1]w^{s-\alpha-1}dw\right|\leqslant 2a^{s-\alpha}/(s-\alpha),\quad \operatorname{Re}s>\alpha,$$

so that from (30) we have the estimate

$$|I_1|\leqslant\sum_{n=1}^{\infty}2g^{s_n^+-\alpha}\,r_n^+/[(s_n^+-\alpha)\,(s_n^+-s)]. \tag{31}$$

Hence it follows that as $g\to 0$

$$\varphi_+(s)=-\frac{b^{s-\alpha}}{K_+(s)}\left[\frac{K_+(s)-K_+(\alpha)}{s-\alpha}+O\left(g^{s_1^+-\alpha}\right)\right]. \tag{32}$$

It can be shown in exactly the same way that

$$\varphi_-(s)=\frac{a^{s-\alpha}}{K_-(s)}\left[\frac{K_-(s)-K_-(\alpha)}{s-\alpha}+O\left(g^{\alpha-s_1^-}\right)\right], \tag{33}$$

where we have denoted the poles and residues in their meromorphic function $K_-(s)$ by s_n^- and r_n^- (it is obvious that all the s_n^- lie on the real axis to the left of the point $s=\alpha$). Hence, we have found the principal terms of the representations for φ_+ and φ_- (we denote them by φ_+° and φ_-°). Taking φ_+° and φ_-° as the zeroth approximation, we can obtain φ_+ and φ_- from (23)-(24) with any required accuracy by iteration. Thus, to a first approximation, using (30), we have

$$\begin{aligned}\varphi_+^1(s)&=-\frac{b^{s-\alpha}}{K_+(s)}\left[\frac{K_+(s)-K_+(\alpha)}{s-\alpha}+\frac{r_1^+K_-(\alpha)g^{s_1^+-\alpha}}{(s_1^+-s)(s_1^+-\alpha)K_-(s_1^+)}\right],\\ \varphi_-^1(s)&=\frac{a^{s-\alpha}}{K_-(s)}\left[\frac{K_-(s)-K_-(\alpha)}{s-\alpha}+\frac{r_1^-K_+(\alpha)\,d^{s_1^--\alpha}}{(s_1^--s)(s_1^--\alpha)K_+(s_1^-)}\right].\end{aligned} \tag{34}$$

We will now determine the elements of the flow. According to (8) we have

$$\begin{aligned}A(s)&=(\varphi_++\varphi_-+\varphi_0)/\sin(\pi\xi/2)=\frac{1}{\sin(\pi\xi/2)}\left[\frac{b^{s-\alpha}-a^{s-\alpha}}{s-\alpha}-\right.\\ &\left.-\frac{K_+(s)-K_+(\alpha)}{(s-\alpha)K_+(s)}b^{s-\alpha}+\frac{a^{s-\alpha}}{K_-(s)}\frac{K_-(s)-K_-(\alpha)}{s-\alpha}\right]=\frac{1}{\sin(\pi\xi/2)}\left[\frac{K_-(s)K_+(\alpha)\,b^{s-\alpha}-K_-(\alpha)K_+(s)\,a^{s-\alpha}}{(s-\alpha)\,K_-(s)K_+(s)}\right]\end{aligned} \tag{35}$$

$$h^*(s,\ \theta)=\frac{\sin(\theta+\pi/2)\,\xi}{\sin(\pi\xi/2)}\left[\frac{K_-(s)K_+(\alpha)b^{s-\alpha}-K_-(\alpha)K_+(s)a^{s-\alpha}}{(s-\alpha)\,K_-(s)\,K_+(s)}\right]. \tag{36}$$

Finally, we obtain

$$\begin{aligned}H(w,\ \theta)&=H_1+h(w,\ \theta)\,w^{\alpha}=H_1+\\ &+\frac{\Delta H}{2\pi i}\int_{\tau-i\infty}^{\tau+i\infty}\frac{\sin(\theta+\pi/2)\,\xi}{2\sin(\pi\xi/2)}\left[\frac{K_-(s)K_+(\alpha)b^{s-\alpha}-K_-(\alpha)K_+(s)a^{s-\alpha}}{(s-\alpha)\,K_-(s)\,K_+(s)}\right]w^{\alpha-s}\,ds,\quad \Delta H=H_2-H_1.\end{aligned} \tag{37}$$

For the strength of the layer of ground R we obtain (the physical characteristics of the flow are established from the equations given in [3])

$$\begin{aligned}R&=\int_a^b kw^{-k-1}H_\theta(w,\ 0)\,dw=\frac{k\Delta H}{2}\frac{1}{2\pi i}\int_{\tau-i\infty}^{\tau+i\infty}\frac{b^{\alpha-k-s}-a^{\alpha-k-s}}{\alpha-k-s}\times\\ &\times\left[\frac{\xi\cos(\pi\xi/2)}{\sin(\pi\xi/2)}\,\frac{K_-(s)\,K_+(\alpha)\,b^{s-\alpha}-K_-(\alpha)\,K_+(s)\,a^{s-\alpha}}{(s-\alpha)\,K_-(s)\,K_+(s)}\right]ds=\\ &=\frac{k\Delta H}{2\pi i}\int_{\tau-i\infty}^{\tau+i\infty}\frac{K_-(s)K_+(\alpha)(b^{-k}-a^{-k}g^{\alpha-s})-K_-(\alpha)K_+(s)\,(b^{-k}g^{s-\alpha}-a^{-k})}{2\,(\alpha-k-s)\,(s-\alpha)}\,ds.\end{aligned} \tag{38}$$

We further have ($\tau = \alpha + \delta$):

$$\frac{1}{2\pi i}\int_{\tau-i\infty}^{\tau+i\infty}\frac{K_-(s)\,K_+(\alpha)}{(\alpha-k-s)\,(s-\alpha)}b^{-k}ds = 0, \quad \frac{1}{2\pi i}\int_{\tau-i\infty}^{\tau+i\infty}\frac{K_-(s)K_+(\alpha)a^{-k}g^{\alpha-s}}{(s-\alpha+k)(s-\alpha)}ds =$$

$$= \frac{K_-(\alpha)\,K_+(\alpha)\,a^{-k}}{k} - \frac{K_-(\alpha-k)K_+(\alpha)\,a^{-k}g^k}{k} + a^{-k}\,K_+(\alpha)\times$$

$$\times\sum_{n=1}^{\infty}\frac{r_n^- g^{\alpha-s_n^-}}{(s_n^- -\alpha)\,(s_n^- -\alpha+k)} = K_+(\alpha)[K_-(\alpha)-K_-(\alpha-k)g^k\,]\,a^{-k}/k+\ldots,$$

$$-\frac{1}{2\pi i}\int_{\tau-i\infty}^{\tau+i\infty}\frac{K_-(\alpha)\,K_+(s)\,b^{-k}\,g^{s-\alpha}}{(\alpha-k-s)\,(s-\alpha)}ds = \sum_{n=1}^{\infty}\frac{K_-(\alpha)\,r_n^+\,b^{-k}\,g^{s_n-\alpha}}{(s_n-\alpha+k)\,(s_n-\alpha)},$$

$$-\frac{1}{2\pi i}\int_{\tau-i\infty}^{\tau+i\infty}\frac{K_-(\alpha)\,K_+(s)\,a^{-k}\,ds}{(s-\alpha+k)\,(s-\alpha)} = -K_-(\alpha)\,K_+(\alpha)\,a^{-k}/k + K_-(\alpha)\,K_+(\alpha-k)\,a^{-k}/k.$$

As a result we obtain (as $g \to 0$)

$$R = \Delta H K_-(\alpha)\,K_+(\alpha-k)\,a^{-k}/2. \tag{39}$$

For the flow rate of the seeping flow we have

$$Q = k\int_a^b w^{-k}H_\theta(w,0)\,dw = \frac{k\Delta H}{2\pi i}\int_{\tau-i\infty}^{\tau+i\infty}\frac{(b^{\alpha-k-s+1}-a^{\alpha-k-s+1})\,[K_-(s)K_+(\alpha)b^{s-\alpha}-K_-(\alpha)K_+(s)a^{s-\alpha}]}{2\,(\alpha-k-s+1)\,(s-\alpha)}ds.$$

This expression is exactly the same as (38), and consequently

$$Q = k\Delta H[K_-(\alpha)K_+(\alpha-k+1) - K_-(\alpha-k+1)\,K_+(\alpha)\,g^{k-1}]\,a^{1-k}/[2\,(k-1)]. \tag{40}$$

Finally, we obtain for the length of the dam L

$$L/2 = \int_b^\infty w^{-k}H_w(w,\ 0)\,dw = (H_1+H_2)/2b^k + k\times$$

$$\times\int_b^\infty [H_1 + w^\alpha h(w,\ 0)]\,w^{-k-1}dw = \Delta H/2b^k + k\int_b^\infty w^{-k-1+\alpha}h(w,\ 0)\,dw,\ \operatorname{Re}s<\alpha-k.$$

or, by the definition of φ_+,

$$L/2 = k\varphi_+(\alpha-k) - \Delta H/2b^k. \tag{41}$$

Note that Eq. (41) is accurate, without using the asymptotic nature of the solution. Substituting (32) into it, we obtain

$$L/2 = \left[\frac{K_+(\alpha-k)-K_+(\alpha)}{K_+(\alpha-k)} - 1\right]\Delta H/2b^k = -\frac{\Delta H K_+(\alpha)}{2b^k\,K_+(\alpha-k)}. \tag{42}$$

Using the last terms of asymptotic (34), we have

$$L/2 = -\frac{\Delta H}{2b^k}\left[\frac{K_+(\alpha)}{K_+(\alpha-k)} + \frac{r_1^+ k K_-(\alpha)}{(s_1^+-\alpha+k)(s_1^+-\alpha)K_+(\alpha-k)\,K_-(s_1^+)}g^{s_1^+-\alpha}\right], \tag{43}$$

where r_1^+ and s_1^+ are given by relations (27). As might have been expected, for large values of d the value of α has only a small effect on the relationship between ΔH, L, and b.

The asymptotics obtained show that for a fairly heavy layer of ground with an accuracy of the order of $g^{s_1^+-\alpha}$ when calculating the flow characteristics in the immediate vicinity of the dam (the velocity distribution, the heads, and the moment of the pressure forces at the base of the dam) one can use the solution of the problem of a dam in an infinite layer of ground (however, when $k \geqslant 1$ this is not true for the discharge – see Eq. (40)). The corresponding distributions were calculated for this case using the solutions obtained above with $\alpha = 0$ (in view of the analogy between the problem of the flow around a groove and the flow under a dam in a layer of ground of infinite depth, one could also use the solution obtained in [4]). The results of the calculation are shown in Fig. 1 for $H_2 = 0$. Here we show the

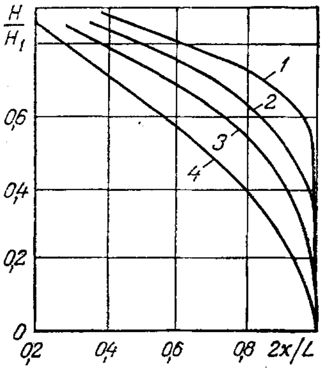

Fig. 1

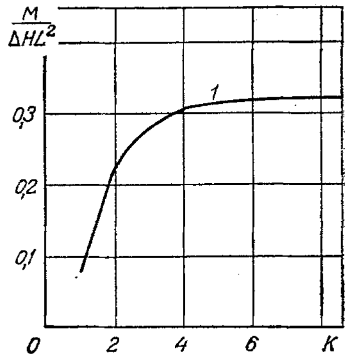

Fig. 2

Fig. 1. Distribution of the head H along the length of the dam for different values of k: 1) k = 8; 2) k = 6; 3) k = 4; and 4) k = 2.

Fig. 2. Curve of the tilting moment of the pressure forces M at the base of the dam with respect to its middle point as a function of k [1) — $f(k) = M/\Delta HL^2$].

distribution of the head over the length of the dam for different values of k, and the tilting moment of the pressure forces at the base of the dam with respect to its middle point (Fig. 2).

NOTATION

H, generalized heat, m; **w**, seepage-rate vector, m/sec; Φ, a dimensionless function describing the seepage law; θ, the angle between the seepage rate vector and the Ox axis, deg; R, the depth of the layer of ground, m; L, the length of the dam, m.

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